Journal of Fluids and Structures (1997) **11,** 447–463

OSCILLATIONS OF ELASTICALLY-MOUNTED CYLINDERS OVER PLANE BEDS IN WAVES

N. Chioukh and R. Narayanan
Department of Civil and Structural Engineering, UMIST, Manchester M60 1QD, U.K.

(Received 22 July 1996 and in revised form 14 January 1997)

The wave-induced oscillations of elastically-mounted horizontal circular cylinders are investigated. The cylinders are well submerged and placed at various distances from a plane bed. The Keulegan–Carpenter number varies to a maximum of 16 and the Reynolds numbers are in the subcritical regime. The cylinders are highly damped. The experimental data reported here include information concerning the transverse oscillations of the cylinders in the presence of regular waves in a laboratory channel. A numerical analysis representing the oscillations of the cylinder as a one-degree-of-freedom system is carried out using an empirical equation for the forcing function relevant to the range of Keulegan–Carpenter numbers of the experiments. The experimental results are compared with the numerical results. The numerical results are also obtained for the experimental conditions of other investigators who carried out experiments with very low damping of the cylinder. The results show the effect of damping on the oscillatory behavior of the cylinder when positioned near a plane bed.

1. INTRODUCTION

The study reported in this paper concerns the transverse vibrations of elastically-mounted rigid cylinders above a plane bed and has applications to submarine pipelines. The amplitudes of transverse oscillations of such cylinders decrease with decreasing gap between the cylinder and the plane bed (Anand & Torum 1985). Tests of Sumer & Fredsøe (1988) with flexibly-mounted cylinders oscillating horizontally in still water, and Anand & Torum (1985) with cylinders in waves, show that the incipient vibrations occur at values of reduced velocity $V_r = U_m f_{nw}/D$ smaller than those in steady flow for small values of Keulegan–Carpenter number $\mathrm{KC} = U_m T_w/D$; U_m and T_w are, respectively, the maximum velocity and period of the oscillating flow, f_{nw} is the natural frequency of the cylinder in water, and D is the cylinder diameter. Comparison between the steady flow and waves for the transverse vibrations of a horizontal cylinder for the same V_r is not strictly valid, because transverse vibration in waves occurs not only from vortex shedding but also from the vertical components of fluid kinematics.

The peak response of a flexible circular cylinder occurs when the ratio of the flow frequency to the cylinder frequency assumes integer values (Bearman & Hall 1987). Sumer & Fredsøe (1988) report multi-peak behavior of transverse vibrations of an elastically-mounted cylinder exposed to an oscillating flow for $\mathrm{KC} > 20$. For $\mathrm{KC} < 7$, no oscillations are recorded.

In this paper experimental results are presented pertaining to the transverse oscillations of three elastically-mounted rigid cylinders exposed to waves in a laboratory flume. The cylinders are horizontal, with their axes parallel to the wave crest. Each of the cylinders is kept above a plane bed with three different gaps. An

0889–9746/97/050447 + 17 \$25.00/fl970088

attempt is made to predict the oscillations as a one-degree-of-freedom system using an empirical equation for the hydrodynamic force proposed by Kao *et al.* (1984).

2. EXPERIMENTAL INVESTIGATION

Experiments were carried out in a wave flume 18·5 m long, 1·2 m wide and 1·0 m deep. Regular waves were generated at one end of the flume by a wave paddle. At the other end was a plywood beach at a slope of 1:10. The maximum reflection coefficient for the beach during the experiments when the cylinder was actually oscillating was about 6%.

The test rig, as shown in Figure 1, consisted basically of two frames, one constrained to oscillate in the vertical direction (frame 1) and the other stationary (frame 2). The test cylinder was smooth and rigid, fixed between two aluminium bars 635 mm apart. The vertical shafts of stainless steel were connected to each aluminnium bar of frame 1, and passed through linear bearings which were fixed to frame 2. The whole unit was placed vertically in the wave flume, and frame 2 was then fixed to a rigid plane supported rigidly on the side walls of the flume. The natural frequency of oscillations

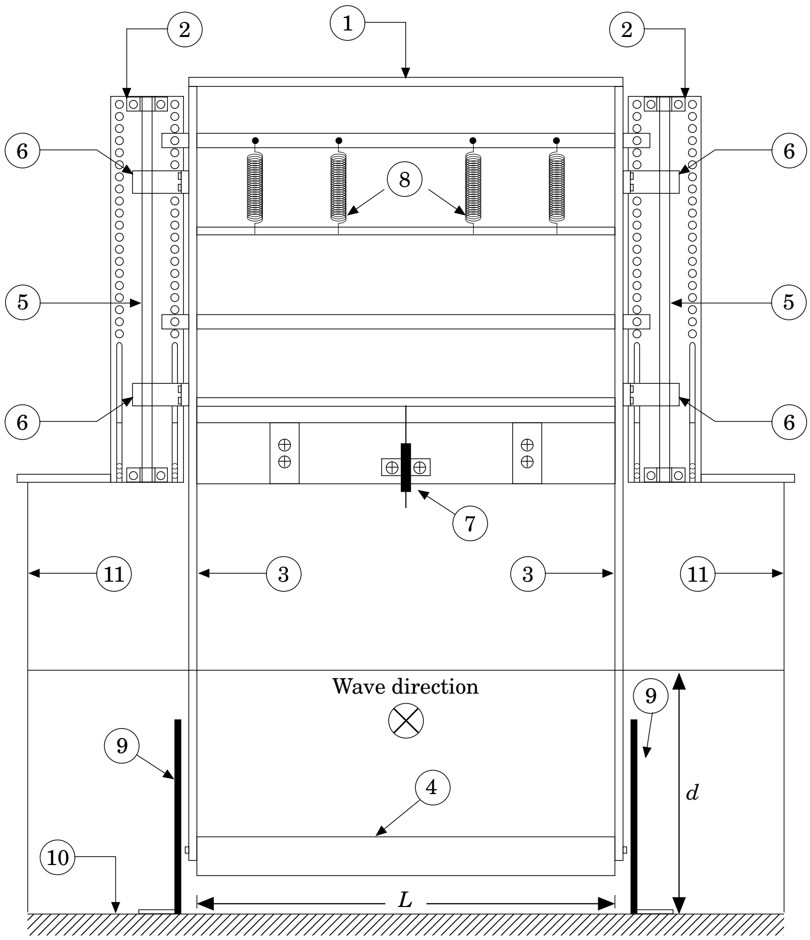

Figure 1. Schematic diagram of the test rig 2. (1)—Frame 1; (2)—Frame 2; (3)—Aluminium bar; (4)—Test cylinder; (5)—Vertical shaft; (6)—Linear ball bearings; (7)—Potentiometer; (8)—Springs; (9)—End plates; (10)—Bed of wave flume; (11)—Walls of flume; d: Water depth; L: Cylinder length.

of frame 1 was governed by linear springs which supported the frame. The springs rested on frame 2. The test cylinder was positioned at a desired distance from the bed of the flume by moving the stationary frame 2 vertically through two slots along its vertical bars. The total mass of the oscillating system was over 7 kg; this large mass is essentially due to the mass of the shafts passing through the linear bearings. Experiments with a pipe of diameter 103 mm, made of high fibre material, were carried out with this arrangement. A few modifications were then made to the apparatus to reduce the mass. The linear bearings were fixed to the oscillating frame 1 and the shafts were attached to frame 2. The mass of the oscillating system was thus reduced by 3 kg. Experiments with pipes of diameter 60 mm and 50 mm were performed on the modified experimental system.

The wave properties were measured by a wave monitor. The wave kinematics were computed using Stokes' higher-order theory. The transverse motions of the test cylinders were sensed by a displacement transducer. The detected signals were digitized and were stored on floppy disks using a PC for further analysis of the data by the mainframe computer of the University of Manchester.

The experimental system consisting of frame 1 with the test cylinder behaved as a one-degree-of-freedom system. The natural frequency and damping of the system were determined both in air and water by the same experimental procedure. The stiffness K was provided by the springs. The experiments in air provided estimates of the natural frequency f_{na} in air and the structural damping ξ_{sa} arising from the bearings as the logarithmic decrement of the free oscillations of the system. The estimated values of f_{na} were in agreement with those calculated simply by using the stiffness K and the mass m_s. The structural characteristics are given in Table 1.

The added mass m_a of water was taken to be $\frac{1}{4}C_{ma}\rho\pi D^2 L$; ρ is the density of water and L is the length of the cylinder. The coefficient C_{ma} as a function of gap ratio G/D has been determined numerically by various investigators [see, for example, Yamamoto *et al.* (1974); Chioukh (1995)] and the chosen values for our study are listed in Table 2. Experiments were carried out in water to determine the system natural frequency f_{nw} and damping ξ_{sw} as a logarithmic decay of free oscillations. The natural frequencies in water for the experimental conditions are presented in Table 2. The heavy structural damping of the system, and the small variations of its value that occurred from one test to another, made it difficult to evaluate the fluid damping accurately. However, it was estimated at $G/D = 1$, and the same value was adopted for the other gaps as well. The fluid damping ξ_w was estimated as $\xi_w = \xi_{sw} - \xi_{sa}$, where ξ_{sa} is the structural damping in air. Values of ξ_w were also estimated by using the analytical expression (Sarpkaya & Isaacson 1981)

$$\xi_w = \frac{1}{M_r}\sqrt{\frac{\pi}{\beta}} + \frac{0{\cdot}34\pi}{4M_r}\left[\frac{A}{D}\right]^2, \tag{1}$$

where A is the amplitude of motion, $M_r = (\text{mass of the system})/(\rho D^2 L)$ and

TABLE 1
Structural characteristics in air

Exp.	D (mm)	m_s (kg)	K (N/m)	ξ_{sa}	f_{na} (Hz)
1	103	7·679	784	0·143	1·57
2	60	7·356	480	0·241	1·35
3	50	4·340	360	0·263	1·45

TABLE 2
Added mass and natural frequency in water

	$G/D = 1$		$G/D = 0{\cdot}398$		$G/D = 0{\cdot}126$	
Exp.	m_a (kg)	f_{nw} (Hz)	m_a (kg)	f_{nw} (Hz)	m_a (kg)	f_{nw} (Hz)
1	5·46	1·25	6·24	1·20	6·88	1·15
2	1·85	1·20	2·11	1·17	2·33	1·13
3	1·29	1·35	1·47	1·30	1·62	1·25

$\beta = f_{nw}D^2/\nu$, ν being the viscosity of the fluid. Using a value of $A/D = 0{\cdot}5$ typical of our experimental results, the calculated values of ξ_w via equation (1), together with the measured values ξ_{sw} and ξ_w, are summarized in Table 3. It is clear that the contribution of ξ_w to ξ_{sw} is relatively small. The stability parameter K_s (Scruton 1963) is defined in this paper without the added mass ($K_s = 4\pi m_s \xi_{sw}/(\rho D^2 L)$) in the same way as was done by Anand & Torum (1985). Such a definition allows important comparison of our experimental and numerical results with their data pertaining to very low damping. In Table 3 are given the values of K_s for the three sets of experiments. The stability parameter K_s includes fluid damping, the measured values of which are given in Table 3.

For the cylinder of diameter 103 mm, the water depth was 515 mm; for the other two cylinders, it was 300 mm. The range of wave heights investigated was 55–280 mm, the Keulegan–Carpenter number KC was 16, and the Reynolds number was of the order of 10^3.

3. ANALYSIS OF THE TRANSVERSE RESPONSE OF THE CYLINDER

For the elastically-mounted cylinder restrained to move only in the transverse direction to the incoming waves, the equation of motion is

$$m_{sw}\ddot{y} + 4\pi f_{nw}\xi_{sw}m_{sw}\dot{y} + 4\pi^2 f_{nw}^2 m_{sw} y = F_y(t), \tag{2}$$

in which $C_{sw} = 4\pi f_{nw}\xi_{sw}m_{sw}$ and $K = 4\pi^2 f_{nw}^2 m_{sw}$; $\ddot{y}, \dot{y}$ and y are, respectively, the acceleration, velocity and displacement in the transverse direction; m_{sw} is the effective mass, which is the sum of the structural mass and the added mass; C_{sw} is the damping, comprising structural and fluid damping; and K is the stiffness of the system. It is not adequate to represent the transverse force with a single frequency; many investigators (Chakrabarti *et al.* 1976; Sarpkaya 1976b; Maull & Norman 1978; Bearman & Hall 1987; Sumer *et al.* 1991) have identified the presence of integer multiples of the flow frequency in transverse force. Chakrabarti *et al.* (1976) presented an equation for the transverse force in terms of a Fourier series, but the coefficients occurring in their

TABLE 3
Damping level

Exp.	ξ_{sw}	ξ_w	ξ_w [equation (1)]	K_g
1	0·175	0·032	0·072	2·73
2	0·225	0·014	0·029	10·30
3	0·318	0·055	0·036	10·93

equation were not for a horizontal cylinder near a plane bed. An alternative expression was proposed by Bearman *et al.* (1984), assuming a quasi-steady vortex street, applicable only to large values of $\mathrm{KC} > 25$. Kao *et al.* (1984) devised an equation based on experiments restricted to $\mathrm{KC} < 20$; they assumed that the transverse force on a horizontal cylinder in waves and located very near to the bed is a combination of the potential flow solutions and the wake effects:

$$F_y(t) = \rho \frac{\pi D^2}{4} L C_{my} \dot{v} + \tfrac{1}{2}\rho D L C_L u^2 + C_w \rho \frac{D}{2} L\left[u^2\left(t - \frac{T_w \theta}{2\pi}\right) - \epsilon u_m^2\right], \qquad (3)$$

where $\dot{v}$ is the vertical acceleration of water particle, and C_w, θ and ϵ are empirical coefficients. The first two terms on the right hand side of the equation are the transverse inertia and lift terms predicted using potential flow theory. The third term is due to the activity of vortex shedding. Kao *et al.* (1984) used experimentally measured forces on a horizontal cylinder placed in waves in a laboratory flume to determine the coefficients C_w, θ and ϵ, expressed as functions of G/D and KC values.

Equations (2) has been solved by an iterative time-stepping procedure, adopting the empirical equation (3) for the forcing function which is appropriate to the present study, restricted to $\mathrm{KC} < 16$. The predicted results encompass our experimental conditions and those of Anand & Torum (1985), and will be discussed after the presentation of experimental results.

4. RESULTS OF THE EXPERIMENTAL INVESTIGATIONS

A large amount of data was collected in this study, but for reasons of brevity only typical results are presented. For more information reference is made to Chioukh (1995). The transverse response of the cylinder is presented in terms of $Y = y - y_{\mathrm{mean}}$ in Figures 2 and 3; y is the instantaneous position of the cylinder with respect to the undisturbed state, and y_{mean} is the mean value of y. For $\mathrm{KC} < 4$ in the majority of cases, the cylinder is found to oscillate more or less at constant amplitude at the wave frequency (Chioukh 1995). As KC increases, the frequency of oscillation of the cylinder is still at the wave frequency, but a second harmonic of the wave frequency begins to assume importance. This is believed to be the result of the activity of weak vortex shedding from the cylinder surface (Sarpkaya 1976b; Williamson 1985a; Bearman 1985; Sumer *et al.* 1991). For $\mathrm{KC} > 7$ (Figures 2 and 3) the oscillatory forces and the vibrations are due mainly to the vortex shedding process interacting in a complicated manner with the cylinder (Bearman 1985; Williamson 1985b; Sumer *et al.* 1991). The correlation length of the vortices along the cylinder which might vary from time to time could possibly contribute to the irregular amplitudes.

When the cylinder is very close to the bed ($G/D = 0{\cdot}126$) it exhibits features similar to those for $G/D = 1$, but the cylinder hits the bed quite severely and the frequency spectra show the occurrence of small spikes at multiple frequencies, perhaps arising from asymmetric vortex shedding due to the small gap.

4.1. Frequency of Oscillations

For the largest diameter of 103 mm, the peak amplitude $Y_{\max}$, expressed nondimensionally with respect to the diameter, is plotted against f_{nw}/f_w in Figures 4 and 5, respectively, for $G/D = 1$ and $G/D = 0{\cdot}126$. In these figures, the ranges of KC are given next to the experimental points. At $G/D = 1$, the amplifications around

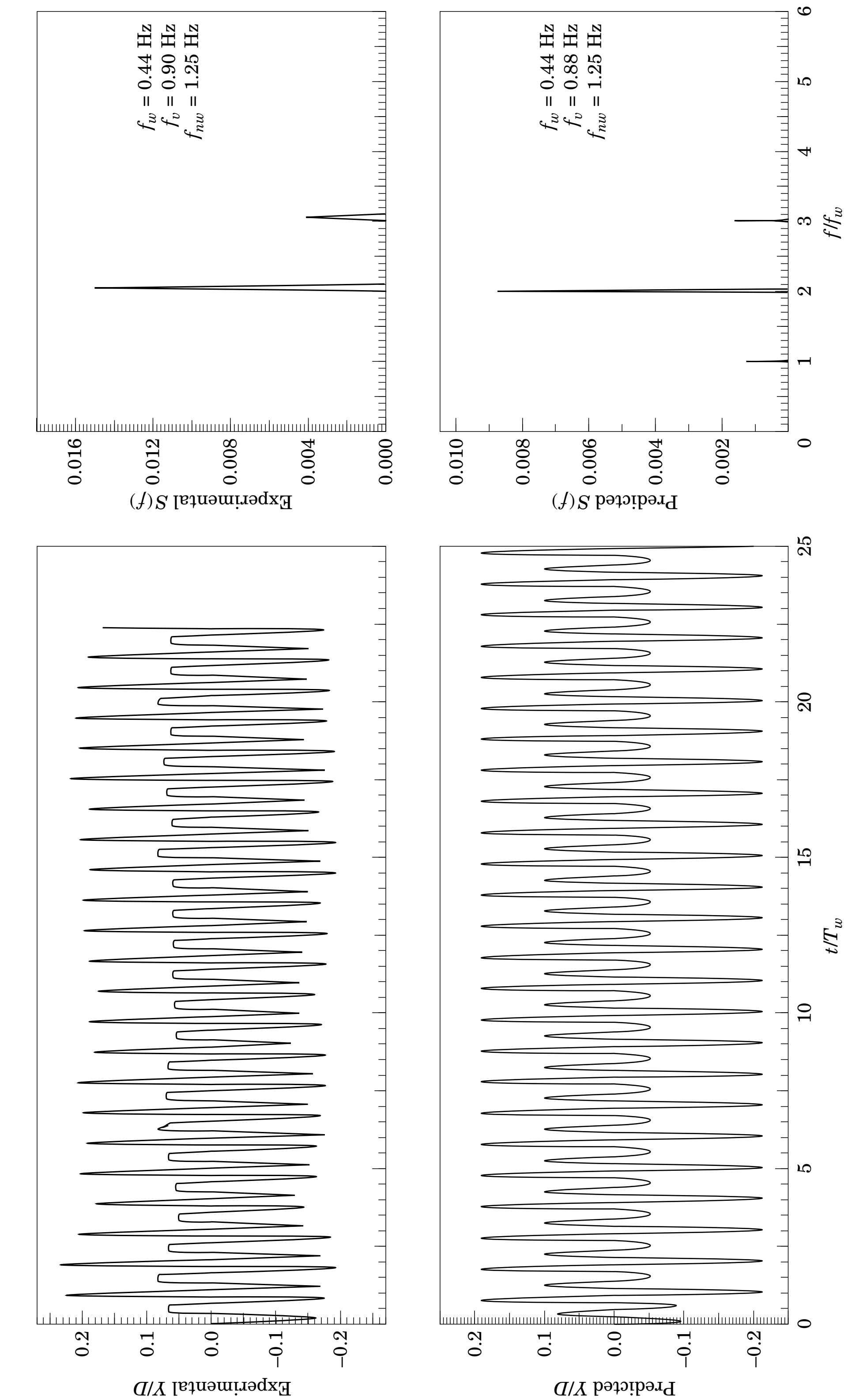

Figure 2. Experimental and predicted transverse response and spectral density for $G/D = 1$, KC = 9·59, $V_r = 3{\cdot}37$, $K_s = 2{\cdot}73$; $D = 0{\cdot}103$ m.

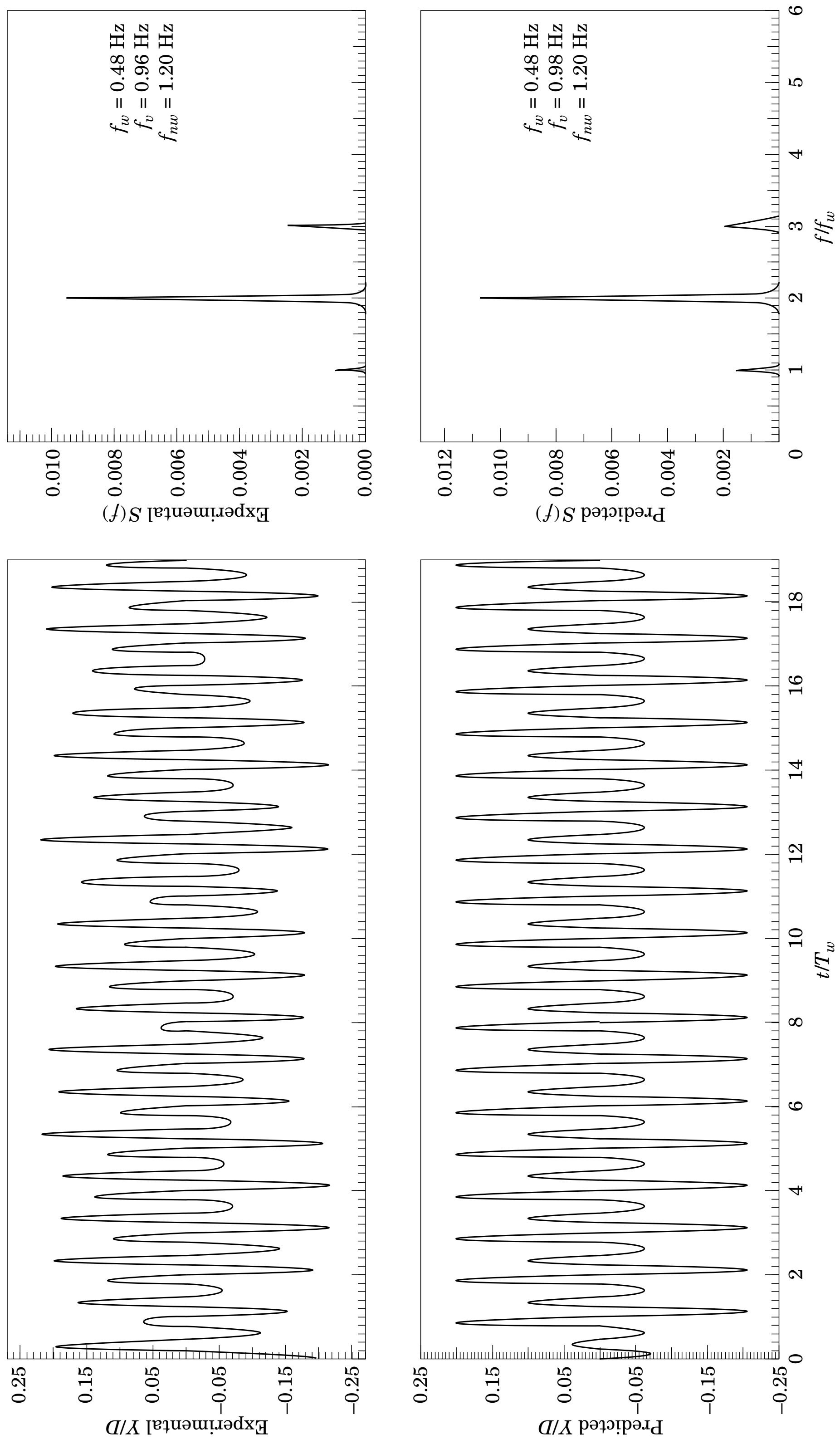

Figure 3. Experimental and predicted transverse response and spectral density for $G/D = 1$, KC = 14·11, $V_r = 5{\cdot}68$, $K_s = 10{\cdot}30$; $D = 0{\cdot}060$ m.

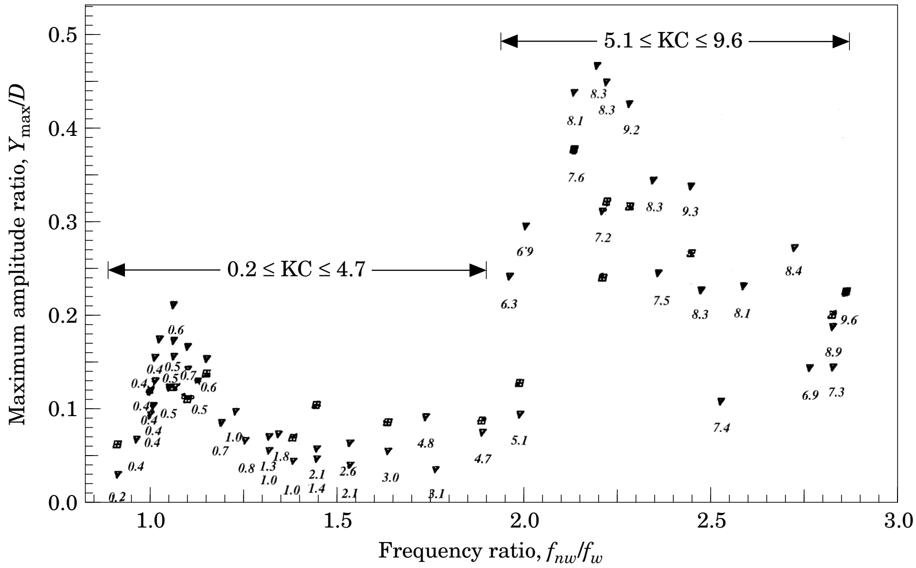

Figure 4. Results of the maximum transverse amplitude versus the frequency ratio f_{nw}/f_w for various KC numbers; $D = 0{\cdot}103$ m, $G/D = 1$, $K_s = 2{\cdot}73$. (▽) Experiments; (□) predictions.

$f_{nw}/f_w = 1$ and 2 are clearly seen. For $G/D = 0{\cdot}126$, the peaks exhibit scatter over a wide range of frequencies. f_{nw}/f_w does not appear to influence the amplitudes for this small value of G/D.

When the ratio of the dominant frequency of oscillations f_v to the wave frequency f_w was plotted against V_r, all the data of the present investigations for $G/D = 1$ collapsed on two nearly constant values of f_v/f_w. There was no clear-cut value of V_r at which transition of f_v from f_w to $2f_w$ occurred. In the range $5 < \mathrm{KC} < 7$, there were some data concerning f_v taking values of f_w and $2f_w$. It was pointed out by Sarpkaya (1976a),

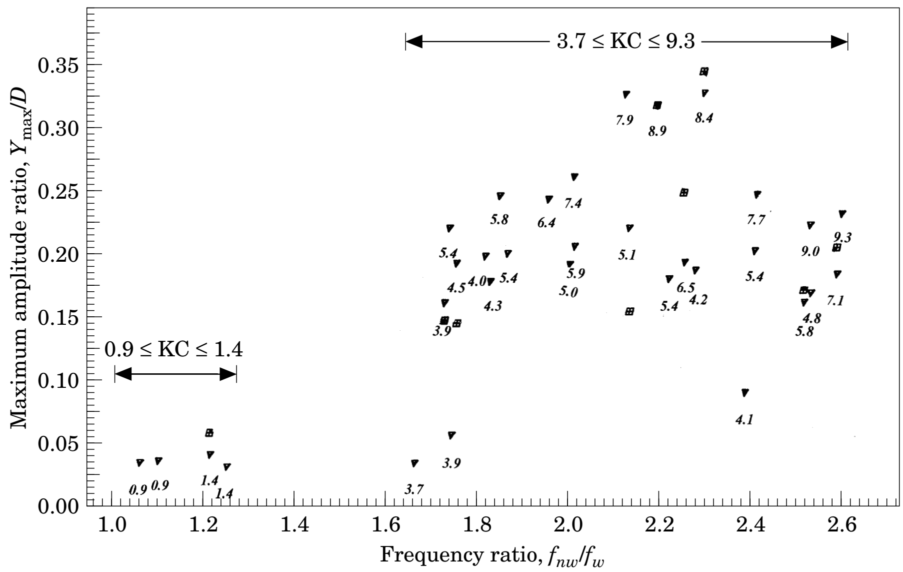

Figure 5. Results of the maximum transverse amplitude versus the frequency ratio f_{nw}/f_w for various KC numbers; $D = 0{\cdot}103$ m, $G/D = 0{\cdot}126$ m, $K_s = 2{\cdot}73$. (▽) Experiments; (□) predictions.

Bearman (1985), Williamson (1985a) and Sumer *et al.* (1991) that occasional asymmetry in the attached vortices begins to appear at around KC = 4, but total asymmetry occurs only at around KC = 7. Perhaps the instabilities of the flow around the cylinder for KC between 4 and 7 make the oscillations meander between two frequencies.

For $G/D = 0{\cdot}126$, the results were found to be similar to those for $G/D = 1$ but the vibrational frequencies at three times the wave frequency appeared at KC around 9. It is not clear how this can happen at this value of KC at which the vortex shedding is expected to occur only at twice the wave frequency (Sarpkaya 1976b; Sumer *et al.* 1991). However, the results of Sumer *et al.* (1991) showed that for a small gap ratio at which the cylinder began to hit the bed, the vibrational frequencies did increase. It is not clear whether $f_v = 3f_w$ is a result of the cylinder hitting the bed or if it is a contribution from the nonlinear waves of finite amplitude that existed in our experiments.

Comparison of the data for $G/D = 1$ and 0·126 suggests that the KC number at which transition of oscillations from the first to the second harmonics of the wave takes place decreases with decreasing gap ratio. Although no measurements relating to vortex shedding were carried out, it is inferred that, as the gap ratio decreases, flow separation and the associated vortex shedding occur at smaller KC numbers. The same conclusion was reached by Chioukh & Narayanan (1994) from their studies of wave forces on stationary cylinders close to a plane bed.

4.2. Mean Position of Oscillations

For the cylinder of diameter 60 mm, the mean position of the gap ratio e/D as a function of V_r is plotted in Figures 6 and 7, respectively, for $G/D = 1$ and $G/D = 0{\cdot}126$. Here e is the mean position of the gap between the cylinder and the bed as the cylinder performs oscillations; it is defined as $e = y_{\text{mean}} + G$. The corresponding KC values are shown next to the points.

At $G/D = 1$, the cylinder did not strike the bed at all. The oscillations are, on

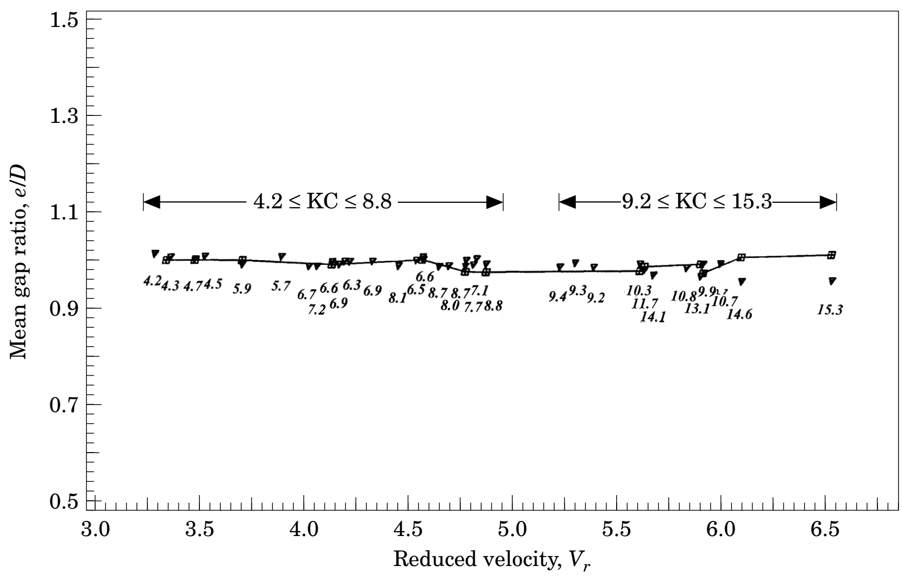

Figure 6. Results of the mean gap versus the reduced velocity for various KC numbers; $D = 0{\cdot}060$ m, $G/D = 1$; $K_s = 10{\cdot}30$. (▽) Experiments; (□) predictions.

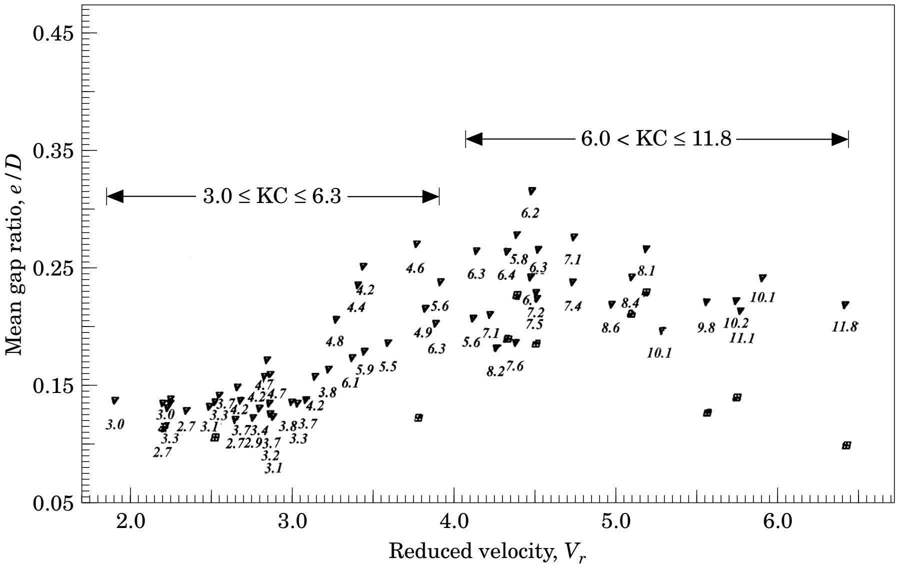

Figure 7. Results of the mean gap versus the reduced velocity for various Kc numbers; $D = 0{\cdot}060$ m, $G/D = 0{\cdot}126$, $K_s = 10{\cdot}30$. (▽) Experiments; (□) predictions.

average, symmetrical (Figure 6), as observed by Anand & Torum (1985) and Sumer *et al.* (1986) for lightly damped cylinders. At $G/D = 0{\cdot}398$, the cylinder hit the bed only occasionally. The mean e/D does not deviate significantly from the undisturbed position. In the experiments of Anand & Torum (1985) asymmetry of oscillations from the undisturbed state was recorded even at $G/D = 0{\cdot}75$. With damping slightly larger than that in the experiments of Anand & Torum (1985), Sumer *et al.* (1986) observed that e/D deviated from the initial position at $G/D = 0{\cdot}4$. Seemingly, the asymmetry of oscillations is initiated at larger G/D as damping of the cylinder is reduced. For $G/D = 0{\cdot}126$ (Figure 7), at $V_r > 2{\cdot}5$ a large drift of the cylinder from the initial position is observed.

4.3. Peak Response of Oscillations

Typical variations of the dimensionless maximum amplitude Y_{max}/D versus KC for the test cylinder of diameter 50 mm are shown in Figures 8 and 9, respectively, for $G/D = 1$ and $G/D = 0{\cdot}126$.

At $G/D = 1$ for $D = 103$ mm ($K_s = 2{\cdot}73$), the maximum value of $Y_{max}/D = 0{\cdot}47$ was recorded around $KC = 8{\cdot}5$, corresponding to the resonant condition. For the cylinder of $D = 50$ mm for which the experiments were conducted at higher K_s and V_r (Figure 8), the results show that Y_{max}/D reaches a maximum value of about 0·28 when KC and V_r assume values around 11 and 5·6, respectively. Anand & Torum (1985) found that $Y_{max}/D = 0{\cdot}7$ at $KC = 10$ for their lightly damped cylinder.

At $G/D = 0{\cdot}398$, our observations show that for $V_r < 4{\cdot}0$ the oscillations do not reach the bed and Y_{max}/D is only slightly less than those for $G/D = 1$. For $D = 50$ mm, large amplitudes of response develop for $V_r = 4{\cdot}2$–$5{\cdot}6$ and $KC = 8$–10. Under these conditions the cylinder hit the bed only occasionally, so that the mean positions are not affected unduly. At the smallest gap, $G/D = 0{\cdot}126$, the cylinder hit the bed most of the

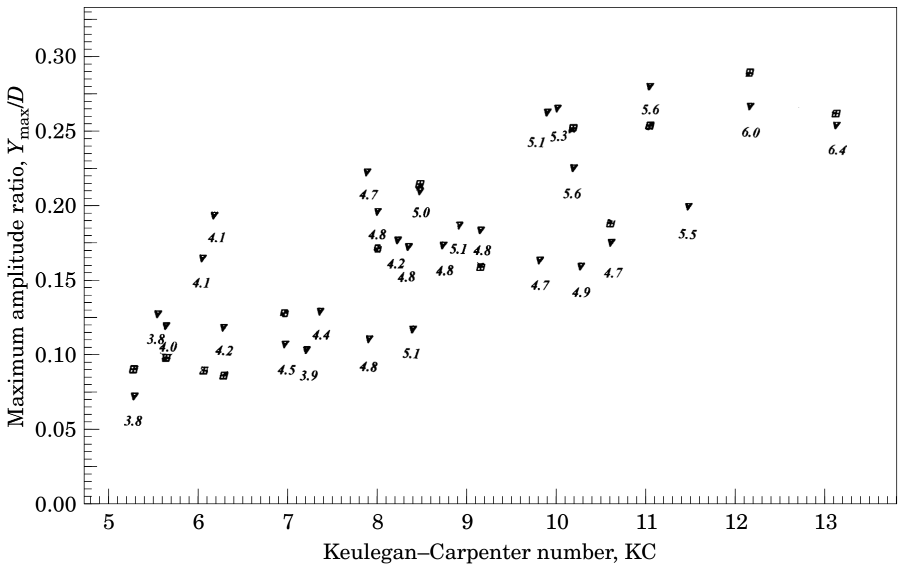

Figure 8. Results of the maximum transverse amplitude versus Keulegan–Carpenter number for various V_r values; $D = 0{\cdot}050$ m, $G/D = 1$, $K_s = 10{\cdot}93$. (▽) Experiments; (□) predictions.

time. The vortex-excited amplitudes appear to initiate at values of KC and V_r smaller than those for $G/D = 1$ and 0·398, and the data exhibit scatter (Figure 9).

4.4. Effects of the Stability Parameter on the Response Amplitudes

Past studies concerning wave-induced vibrations of horizontal cylinders have been for smaller values of stability parameters ($K_s \leq 1{\cdot}5$). A few tests were carried out by Sumer *et al.* (1986) for $K_s = 3{\cdot}5$ at $\mathrm{KC} = 40$. They found the normalized double amplitude ($2A/D$), measured from crests to troughs and averaged over many cycles, to decrease

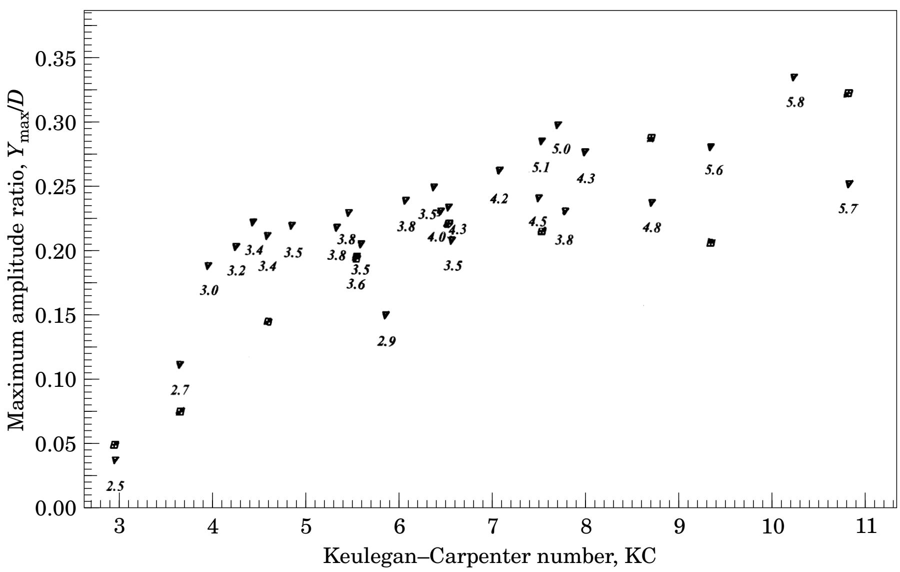

Figure 9. Results of the maximum transverse amplitude versus Keulegan–Carpenter number for various V_r values; $D = 0{\cdot}050$ m, $G/D = 0{\cdot}126$, $K_s = 10{\cdot}93$. (▽) Experiments; (□) predictions.

TABLE 4
Comparison with other experimental results

Experiments (wave flows)	K_s	G/D	Y_{max}/D
		1	0·70
Anand & Torum (1985)	0·18	0·75	0·63
		0·50	0·44
		1	0·47
Present	2·73	0·398	0·42
		0·126	0·33
		1	0·28
Present	10·30	0·398	0·62
		0·126	0·33
		1	0·27
Present	10·93	0·398	0·65
		0·126	0·34

for all V_r values when K_s increased from 1·5 to 3·5. In steady flows, studies were carried out for K_s values as high as 8·84 (Torum *et al.* 1989). In Table 4 are summarized the results of the peak amplitudes Y_{max}/D for $8 \leq KC \leq 10$ and $4 \leq V_r \leq 6$, for different values of G/D and K_s, and compared with the results of Anand & Torum (1985).

It is seen that for $G/D = 1$, the effects of increasing values of K_s are to decrease the peak amplitude Y_{max}/D in accordance with Sumer *et al.* (1986) for KC = 40. It should be mentioned that in the experiments of Sumer *et al.* (1986) it was the mass of the system which was varied and not the damping. However, for the smaller gap ratios ($G/D \leq 0{\cdot}5$) at which the cylinders hit the bed, Y_{max}/D is not greatly affected by the increased values of K_s. This is again in agreement with the results of Sumer *et al.* (1986). It appears that the stability parameter presents only second-order effects with respect to the amplitudes when the cylinders are performing oscillations close to the bed. Similar trends were observed in the experiments of Torum *et al.* (1989) pertaining to cylinders oscillating in steady currents. The reason for this behavior is not clear.

5. PREDICTED RESULTS AND COMPARISON WITH EXPERIMENTS

5.1. COMPARISON WITH THE PRESENT EXPERIMENTS (LARGE K_s)

The predicted results from the theoretical model for a few selected experimental conditions are presented on the same figures (Figures 2 and 3), along with the experimental results discussed earlier. The time histories of the responses and their frequency spectra are well predicted qualitatively. The major changes that occur in the responses and their spectra, when the flow changes from a potential flow to the region where vortex shedding is dominant, are well represented. However, the spectral energies at all the frequencies are not always well evaluated.

The experimental data of the peak amplitudes Y_{max}/D are well predicted for all KC numbers less than 4 or 5 (Figures 4 and 5) at which the potential flow contributions in equation (3) are dominant even when the cylinder is oscillating. At $G/D = 1$, the experimental results pertaining to y_{max}/D (Figure 4) and mean gap ratio e/D (Figure 6) compare well with the predictions. The small differences between the measurements and the predictions are possibly due to the fact that the force expressed by equation (3) is strictly applicable to stationary cylinders. There is, however, evidence that the force increases when the cylinder oscillates (Williamson 1985b; Borthwick & Herbert 1988; Sumer *et al.* 1993). For larger K_c numbers, predictions of $y_{\max}/D$ and e/D (Figures 5 and 7) deteriorate for decreasing gap ratios. It is believed that at smaller gap ratios for the oscillating cylinder striking the bed, the transverse forces are not well represented by equation (3).

5.2. Comparison With Other Experiments (Small K_s)

For $G/D = 1$ and f_{nw} close enough to the vortex shedding frequency, the frequency spectra of the transverse force $F_y(t)$, given by equation (3), and the normalized transverse response (Y/D) are shown in Figure 10, respectively, for damping ratios ξ_{sw} of 0·0095 ($K_s = 0{\cdot}54$), 0·00633 ($K_s = 0{\cdot}36$), and 0·00317 ($K_s = 0{\cdot}18$). $\xi_{sw} = 0{\cdot}00317$

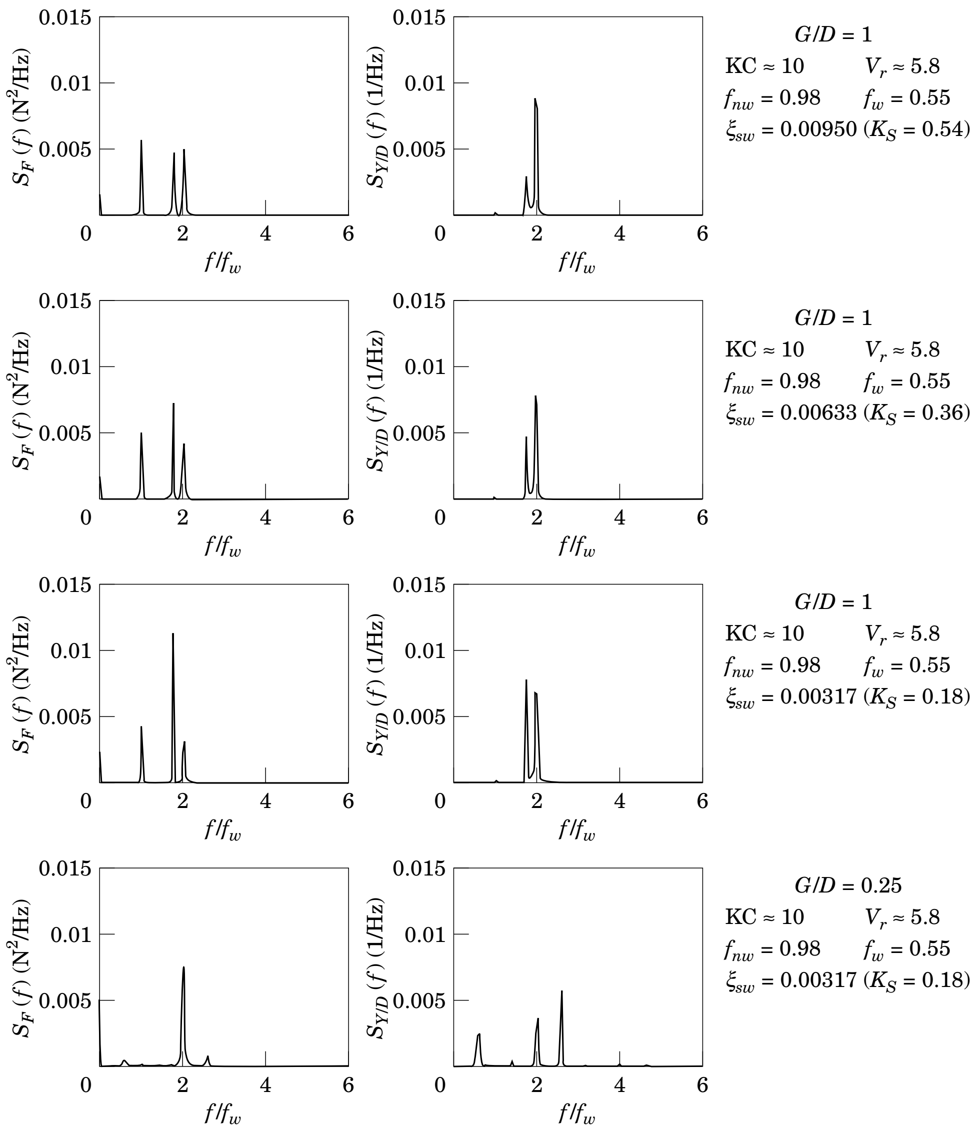

Figure 10. Predicted spectra of wave forces and cylinder transverse responses.

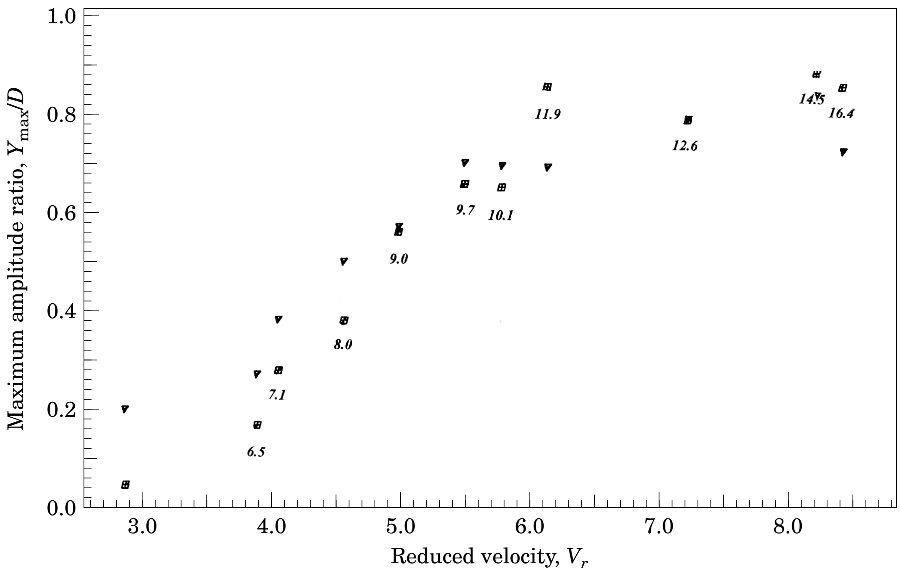

Figure 11. Results of the maximum transverse amplitude versus the reduced velocity for various KC numbers: (▽) experimental data of Anand & Torum (1985); (□) present numerical predictions; $D = 0{\cdot}050$ m, $G/D = 1$, $K_s = 0{\cdot}180$.

($K_s = 0{\cdot}18$) is the value of damping typical of the experiments of Anand & Torum (1985). It is seen that as the damping is decreased, the main vibratory frequency of the force and the response tend to shift from the vortex shedding frequency to the natural frequency f_{nw} of the system. This indicates that the lock-in mechanism is possible only for very small damping, as reported by King (1977) for steady flows. However, when G/D is reduced to 0·25 (Figure 10) the cylinder is found to hit the bed, and the frequency of the force is re-established with the vortex shedding frequency, whereas that of the response is higher. The contribution at f_{nw} is no longer present. Perhaps as the cylinder hits the bed, part of its movement is arrested, resulting in a frequency higher than that in the situation of no impact. This gives the impression that, at small gap ratios where the cylinder hits the bed, the response becomes controlled neither by the vortex shedding frequency nor by the natural frequency of the system, but by another frequency higher than the vortex shedding frequency. This result seems to be consistent with that of Sumer *et al.* (1986).

The peak amplitudes Y_{max}/D observed by Anand & Torum (1985) are well predicted by the analytical model only for $G/D = 1$ (Figure 11). At a small gap ratio ($G/D = 0{\cdot}5$, Figure 12), when the cylinder hits the bed at all times, most of the experimental values do not agree with the predictions.

The effects of increased damping on the oscillatory amplitudes are further investigated for one case ($K_c \approx 10$ and $V_r \approx 5{\cdot}8$) at which the experimental values of Y_{max}/d due to Anand & Torum (1985) are well predicted for all three gap ratios. The results of this analysis, in which the damping ratio (ξ_{sw}) is varied from 0·00317 ($K_s = 0{\cdot}18$) to 0·2639 ($K_s = 15$), are shown in Figure 13. The present experimental data are also shown in the figure and are vertical lines representing wide scatter of results at KC = 10 and $V_r = 5{\cdot}8$. It should be noted that, for the sake of comparison, our experimental data for $G/D = 0{\cdot}398$ are grouped with the data of Anand & Torum (1985) pertaining to $G/D = 0{\cdot}5$. It is seen that for $G/D = 1$ and 0·75 the maximum response amplitudes observed in our experiments is dependent on K_s. The predictions for $G/D = 1$ are in

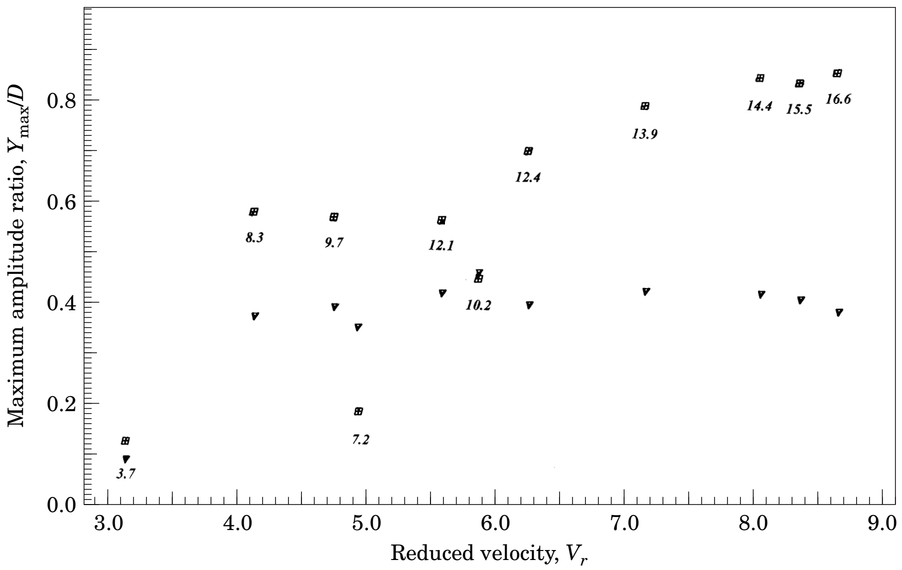

Figure 12. Results of the maximum transverse amplitude versus the reduced velocity for various KC numbers: (▽) experimental data of Anand & Torum (1985); (□) present numerical predictions; $D = 0{\cdot}050$ m, $G/D = 0{\cdot}5$, $K_s = 0{\cdot}180$.

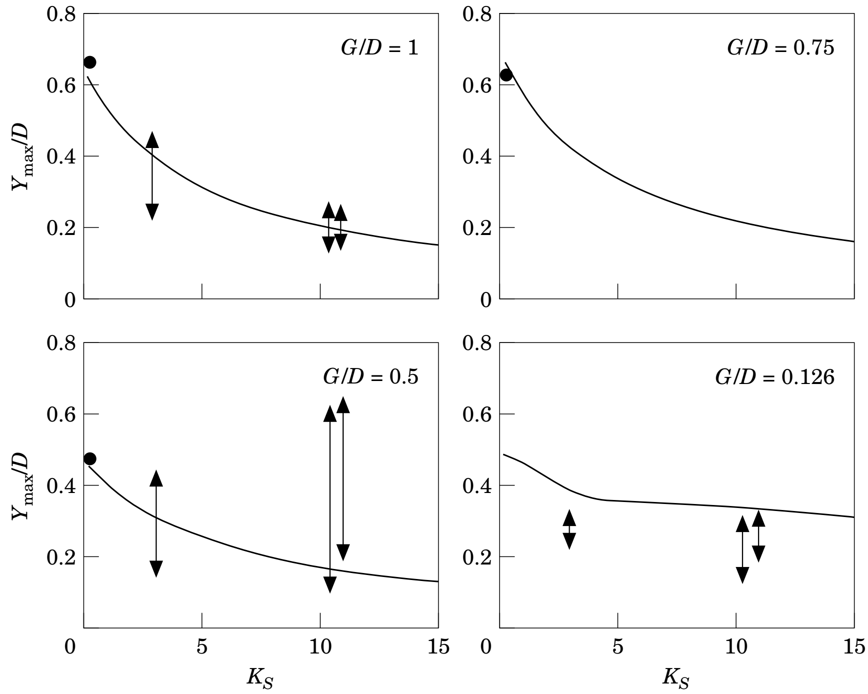

Figure 13. Maximum amplitude of the transverse oscillations as a function of the stability parameter, for KC ≃ 10, $V_r \simeq 5{\cdot}8$, $\xi_{sw} = 0{\cdot}00317$–$0{\cdot}2639$: ——, predicted; (●) experimental from Anand & Torum (1985); (↕) present experimental data.

reasonable agreement with the measurements. However, the present data for $G/D = 0{\cdot}5$ at larger values of K_s show considerable scatter, particularly because large amplitudes occur only occasionally in the tests. For $G/D = 0{\cdot}126$, although the results are not well predicted, the general trend of the theoretical curve agrees with the measurements, showing that, for $K_s > 2{\cdot}5$, $Y_{\max}/D$ is less dependent on K_s.

6. CONCLUSIONS

Wave-induced transverse vibrations of horizontal circular cylinders placed over a plane bed were studied both experimentally and theoretically. The main conclusions of this investigation are as follows.

1. For very small KC numbers at which vortex shedding does not take place, the responses fluctuate with regular amplitudes and with the vibratory frequency equal to the wave frequency. The responses amplify when the frequency ratio $f_{nw}/f_w = 1$.

2. For the larger KC numbers, the response amplitudes exhibit irregularity. For large gap ratios at which the cylinder does not reach the bed during oscillations, the peak response amplitudes correlate better with f_{nw}/f_w than with KC. For small gap ratios at which the cylinder touches the bed during oscillations, the amplitudes do not show strong dependence on f_{nw}/f_w, KC or V_r.

3. When the cylinder is away from the bed the responses are symmetrical. But as the oscillations begin to reach the bed the cylinder is repelled, leading to amplification of the amplitudes in the upward direction.

4. At gap ratios where the cylinder does not interact with the bed, the main frequencies of the oscillations are the same as the vortex shedding frequencies. As the cylinder approaches the bed, transition of the vibratory response from f_w to $2f_w$ occurs at smaller KC numbers.

5. The effect of increasing the stability parameter K_s on the response amplitudes is to reduce their magnitude when the cylinder does not hit the bed. The amplitudes seem to be less affected by the magnitude of the stability parameter when the undisturbed position of the cylinder is very close to the bed.

6. The theoretical model has shown that for cylinders away from the bed the peak amplitudes could be reasonably estimated, provided that the force coefficients are well selected.

ACKNOWLEDGEMENTS

The authors wish to thank Messrs Les Bray and Steve Edwards for their help with the experimental apparatus and instrumentation. Thanks are also due to Dr Mustafa Gunal for his help in improving the clarity of the figures.

REFERENCES

ANAND, N. M. & TORUM, A. 1985 Free span vibrations of submarine pipelines in steady flow and waves. In *Proceedings International Symposium on Separated Flow around Marine Structures,* Trondheim, Norway, pp. 155–199.

BEARMAN, P. W. 1985 Vortex trajectories in oscillatory flow. In *Proceedings International Symposium around Marine Structures,* Trondheim, Norway, pp. 133–153.

BEARMAN, P. W. & HALL, P. F. 1987 Dynamic response of circular cylinders in oscillatory flow and waves. In *Proceedings of International Conference on Flow-Induced Vibrations,* Bowness-on-Windermere, U.K. pp. 183–190 Cranfield: BHRA.

BEARMAN, P. W., GRAHAM, J. M. R. & OBASAJU, E. D. 1984 A model equation for the transverse forces on cylinders in oscillatory flows. *Applied Ocean Research* **6,** 166–172.

Borthwick, A. G. L. & Herbert, D. M. 1988 Loading and response of a smaller diameter flexibly mounted cylinder in waves. *Journal of Fluids and Structures* **2,** 479–501.

Chakrabarti, S., Wolbert, A. L. & Tam, W. A. 1976 Wave forces on vertical circular cylinder. *ASCE Journal of the Waterways, Harbors and Coastal Engineering Division* **102,** 203–221.

Chioukh, N. 1995 Wave effects on rigid and elastically-mounted horizontal circular cylinders placed above a plane bed. PhD thesis, University of Manchester Institute of Science and Technology, Manchester, U.K.

Chioukh, N. & Narayanan, R. 1994 Inertia dominated forces on oblique horizontal cylinders in waves near a plane boundary. *Journal of Coastal Engineering* **22,** 185–199.

Kao, C. C., Daemrich, K. F., Kohlbase, S. & Partenscky, H. W. 1984 Transverse force due to wave on cylinder near bottom. In *Symposium on Ocean Structures Dynamics,* Corvallis, Oregon, USA, pp. 356–368.

King, R. 1977 A review of vortex shedding research and its applications. *Journal of Ocean Engineering* **4,** 141–171.

Maull, D. J. & Norman, S. G. 1978. A horizontal circular cylinder under waves. In *Proceedings of Mechanics of Wave-Induced Forces on Cylinder* (ed. T. L. Shaw), Bristol, U.K., pp. 359–378.

Sarpkaya, T. 1976a Vortex shedding and resistance in harmonic flow about smooth and rough circular cylinders at high Reynolds numbers. Naval Postgraduate School of Monterey, Report No. NPS-59SL76021, California, U.S.A.

Sarpkaya, T. 1976b Forces on cylinders near a plane boundary in a sinusoidally oscillating Fluid. *ASME Journal of Fluids Engineering* **98,** 499–505.

Sarpkaya, T. & Isaacson, M. 1981 *Mechanics of Wave Forces on Offshore Structures.* New York: Van Nostrand Reinhold.

Scruton, C. 1963 On the wind excited oscillation of stacks, towers and masts. In *Proceedings of the Conference on Wind Effects on Buildings and Structures,* National Physical Laboratory, Teddington, England.

Sumer, B. M. & Fredsøe, J. 1988 Effect of Reynolds number on vibration of cylinders. *ASME Journal of Offshore Mechanics and Arctic Engineering* **111,** 131–137.

Sumer, B. M., Fredsøe, J. & Jacobsen, V. 1986 Transverse vibration of pipeline exposed to waves. In *Proceedings International Conference on Offshore Mechanics and Arctic Engineering,* Tokyo, Japan, Vol. III, pp. 588–596.

Sumer, B. M. & Fredsøe, J. 1988. Transverse vibrations of an elastically mounted cylinder exposed to an oscillating flow. *ASME Journal of Offshore Mechanics and Arctic Engineering* **110,** 387–394.

Sumer, B. M., Jensen, B. L. & Fredsøe, J. 1991 Effect of plane boundary on oscillating flow around a circular cylinder. *Journal of Fluid Mechanics* **225,** 271–230.

Sumer, B. M., Fredsøe, J., Jensen, B. L. & Christiansen, N. 1993 Forces on vibrating cylinder near wall in current and waves. *ASCE Journal of the Waterway, Port, Coastal and Ocean Engineering* **120,** 233–250.

Torum, A., Bostrom, B. & Sandey, A. B. 1989 On the damping of and forces on a current induced vibrating pipeline. In *Proceedings International Conference on Offshore Mechanics and Arctic Engineering,* The Hague, pp. 319–326.

Williamson, C. H. K. 1985a Sinusoidal flow relative to circular cylinders. *Journal of Fluid Mechanics* **155,** 141–174.

Williamson, C. H. K. 1985b In-line response of a cylinder in oscillatory flow. *Applied Ocean Research* **7,** 97–106.

Yamamoto, T., Nath, J. H. & Slotta, L. S. 1974 Wave forces on cylinders near plane boundary. *ASCE Journal of the Waterway, Port, Coastal and Ocean Engineering* **100,** 345–359.